MTH301 Calculus

Final Term Examination – Spring 2005

Time Allowed: 150 Minutes

Please read the following instructions carefully before attempting any of the questions:

1-Duration of the paper is 150 minutes and use of calculator is allowed

2. Attempt all questions. Marks are written adjacent to each question.

3. Do not ask any questions about the contents of this examination from anyone.

a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.

b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

c. Write all steps, missing steps may lead to deduction of marks.

4- Pasting the equations of math type from word file into software may cause some visibility problem, so please note that do not copy equations of math type into software from word file. Paste the equations from math type directly into software.

Total Marks: 65 **Total Questions: 12**

Question No. 1 **Marks : 08**

Evaluate the iterated integral

$$\iint_R x\sqrt{(1-x^2)}\,dA \qquad R=\{(x,y):0\le x\le 1, 2\le y\le 3\}$$

Question No. 2 **Marks : 06**

Find the directional derivative $D_{\vec{u}} f(-2,0)$ of $f(x,y) = x^2 - 3xy + 4y^3$ in the direction of $\vec{u} = \hat{i} + 2\hat{j}$ at the point (-2, 0).

Question No. 3 **Marks : 05**

Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 8x + 6y + 12z = 3$.

Question No. 4 **Marks : 02**

Let $\vec{a} = \frac{4}{3}\hat{i} + \hat{j}$ and $\vec{b} = 4\hat{i} + 3\hat{j}$, then

- $\vec{a}$ and $\vec{b}$ are orthogonal
- The angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$.
- $\vec{a}$ and $\vec{b}$ are parallel.
- The angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$.

Question No. 5 **Marks : 02**

The Laplace transform of e^{2t} is

- $\frac{1}{s^2 + 4}$ if $s > 2$
- $\frac{1}{s^2 - 4}$ if $s > 2$
- $\frac{1}{s + 2}$ if $s > 2$
- $\frac{1}{s - 2}$ if $s > 2$

Question No. 6 **Marks : 02**

If (4, $\frac{\pi}{3}$,- 4) is a point in the Cylindrical coordinates then the same point in Rectangular coordinates is given by

- (2,2$\sqrt{3}$,4)
- (2,2$\sqrt{3}$,4)
- (2,2$\sqrt{3}$,-4)
- (- 2,2$\sqrt{3}$,4)
- (2$\sqrt{3}$,2,-4)

Question No. 7 **Marks : 02**

The differential $dz = Mdx + Ndy$ is an exact differential if we have

- $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$
- $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
- $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$
- $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$

Question No. 8 **Marks : 10**

Compute Curl and Divergence for the given vector field $\vec{F} = x^2\hat{i} + 4xy^3\hat{j} + y^2x\hat{k}$.

Question No. 9 **Marks : 02**

The function $y = x^{2/3} + 6$ is

- Even function
- Odd function
- Neither even nor odd
- Constant

Question No. 10 **Marks : 06**

Determine whether the differential $dz = (xlny + xy)\,dx + (ylnx + xy)\,dy$ is exact or not?

Question No. 11 **Marks : 10**

Determine $L^{-1}\{\frac{s-19}{s^2+3s-10}\}$ by using the concept of partial fraction.

Question No. 12 **Marks : 10**

Locate all the relative maxima, relative minima and saddle points for function

$f(x, y) = x^2 + y^2 - x^2y$